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SUPPLEMENT TO
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FLUTTER, VIBRATION, AND BUCKLING OF TRUNCATED ORTHOTROPIC CONICAL SHELLS WITH GENERALIZED ELASTIC EDGE RESTRAINT

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INTRODUCTION

An analysis of the flutter, vibration, and buckling of orthotropic conical shells is presented in NASA TN D-5759. Certain lengthy expressions which were developed in the analysis are required for complete documentation but were considered to be of interest to only a limited group of readers. This supplement to NASA TN D-5759 presents these expressions in four sections. The first section gives the details of the solution of the compatibility equations. The second section presents the expressions l_{ij}, g_{ij}, h_{ij}, and $\bar{e}_{ij}$ that appear in the boundary conditions for the stress functions. The third section presents expressions for the matrix elements b_{ij}, c_{ij}, d_{ij}, e_{ij}, and f_{ij} of the governing stability equation (eq. (8)). The fourth section presents expressions for the nondimensional parameters S_{ij} and M_{ij} which relate the stiffness and inertia characteristics of end rings to the ring and shell material properties and geometry. The parameters S_{ij} and M_{ij} appear in the expressions presented in the second and third sections. The notation form $S_{ij,1}$ refers to the small end of the conical shell and $S_{ij,2}$ to the large end. All symbols not defined herein are defined in the list of symbols in NASA TN D-5759.

SOLUTION OF COMPATIBILITY EQUATION

The compatibility equation is

$$\left(y\epsilon_{y\theta B,\phi}\right)_{,y} - \epsilon_{yB,\phi\phi} - \left(y^2\epsilon_{\theta B,y}\right)_{,y} + y\epsilon_{yB,y} = y \cot \alpha \; w_{B,yy} \tag{S1}$$

or, in terms of the stress function F,

$$L_2(F) = -B_\theta\left(1 - \mu'_\theta\mu'_y\right)w_{B,yy} \cot \alpha \tag{S2}$$

where

$$L_2(\;) = y(\;)_{,yyyy} + \frac{B_\theta}{B_y}\left[\frac{1}{y^3}(\;)_{,\phi\phi\phi\phi} + \frac{2}{y^3}(\;)_{,\phi\phi} + \frac{1}{y^2}(\;)_{,y} - \frac{1}{y}(\;)_{,yy}\right]$$

$$+ \left[\frac{B_\theta}{B_{y\theta}}\left(1 - \mu'_\theta\mu'_y\right) - 2\mu'_\theta\right]\left[\frac{1}{y^3}(\;)_{,\phi\phi} - \frac{1}{y^2}(\;)_{,y\phi\phi} + \frac{1}{y}(\;)_{,yy\phi\phi}\right] \tag{S3}$$

Equation (S2) has variable coefficients that reduce to constant coefficients by use of the transformation of coordinates

$$y = y_1 e^x \tag{S4}$$

For simple harmonic motion the displacement w_B is assumed to be

$$w_B = w_B^*(x) \cos n\phi \; e^{i\omega t} \tag{S5}$$

where

$$w_B^*(x) = a_o + \bar{a}_o + \sum_{m=1}^{M} a_m \sin \frac{m\pi x}{x_1} \tag{S6}$$

If the stress function F is taken to be

$$F = e^x f(x) \cos n\phi \; e^{i\omega t} \tag{S7}$$

and equations (S4) and (S6) are utilized, equation (S2) reduces to the ordinary differential equation

$$f_{,xxxx} + P_1 f_{,xx} + P_2 f = -B_\theta\left(1 - \mu'_\theta\mu'_y\right)y_1 \cos \alpha \left(w^*_{B,xx} - w^*_{B,x}\right) \tag{S8}$$

where

$$\left.\begin{aligned} P_1 &= -\left(1 + \frac{B_\theta}{B_y}\right) - n^2\left[\frac{B_\theta}{B_{y\theta}}\left(1 - \mu'_\theta\mu'_y\right) - 2\mu'_\theta\right] \\ P_2 &= \frac{B_\theta}{B_y}\left(n^2 - 1\right)^2 \end{aligned}\right\} \tag{S9}$$

By the use of equation (S7), equation (S8) contains only even derivatives of f with respect to x. The homogeneous solution is readily found to be

$$f_h = \bar{A}_1 e^{\lambda_1 x} + \bar{A}_2 e^{-\lambda_1 x} + \bar{A}_3 e^{\lambda_2 x} + \bar{A}_4 e^{-\lambda_2 x} \tag{S10}$$

where

$$\left.\begin{aligned} \lambda_1 &= \left[\frac{-P_1 + \left(P_1^2 - 4P_2\right)^{1/2}}{2}\right]^{1/2} \\ \lambda_2 &= \left[\frac{-P_1 - \left(P_1^2 - 4P_2\right)^{1/2}}{2}\right]^{1/2} \end{aligned}\right\} \tag{S11}$$

The particular solution is obtained by substituting

$$f_p = d_o + \sum_{m=1}^{M}\left(d_m \cos\frac{m\pi x}{x_1} + e_m \sin\frac{m\pi x}{x_1}\right) \tag{S12}$$

into equation (S8) and matching coefficients; this gives

$$\left.\begin{aligned} d_o &= \frac{B_\theta\left(1 - \mu'_\theta\mu'_y\right)y_1 \cot\alpha}{P_2}\bar{a}_o \\ d_m &= \frac{B_\theta\left(1 - \mu'_\theta\mu'_y\right)y_1 \cot\alpha}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\frac{m\pi}{x_1}a_m \\ e_m &= \frac{B_\theta\left(1 - \mu'_\theta\mu'_y\right)y_1 \cot\alpha}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\left(\frac{m\pi}{x_1}\right)^2 a_m \end{aligned}\right\} \tag{S13}$$

Thus $f = f_h + f_p$ becomes

$$f = B_\theta\left(1 - \mu'_\theta\mu'_y\right)y_1 \cot\alpha \left\{A_1 e^{\lambda_1 x} + A_2 e^{-\lambda_1 x} + A_3 e^{\lambda_2 x} + A_4 e^{-\lambda_2 x} + \frac{\bar{a}_o}{P_2}\right.$$

$$\left. + \sum_{m=1}^{M} \frac{\left[\left(\frac{m\pi}{x_1}\right)^2 \sin\frac{m\pi x}{x_1} + \frac{m\pi}{x_1}\cos\frac{m\pi x}{x_1}\right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2} a_m\right\} \tag{S14}$$

The stress resultants become

$$N_{\theta B} = \frac{B_\theta\left(1 - \mu'_\theta\mu'_y\right)\cot\alpha}{y_1 e^x}\left\{A_1\lambda_1\left(1 + \lambda_1\right)e^{\lambda_1 x} - A_2\lambda_1\left(1 - \lambda_1\right)e^{-\lambda_1 x} + A_3\lambda_2\left(1 + \lambda_2\right)e^{\lambda_2 x}\right.$$

$$\left. - A_4\lambda_2\left(1 - \lambda_2\right)e^{-\lambda_2 x} - \sum_{m=1}^{M} \frac{\left[\left(\frac{m\pi}{x_1}\right)^4 + \left(\frac{m\pi}{x_1}\right)^2\right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2} \sin\frac{m\pi x}{x_1}\, a_m\right\} \cos n\phi \tag{S15}$$

$$N_{yB} = \frac{B_\theta\left(1 - \mu'_\theta\mu'_y\right)\cot\alpha}{y_1 e^x}\left(A_1\left(1 + \lambda_1 - n^2\right)e^{\lambda_1 x} + A_2\left(1 - \lambda_1 - n^2\right)e^{-\lambda_1 x}\right.$$

$$+ A_3\left(1 + \lambda_2 - n^2\right)e^{\lambda_2 x} + A_4\left(1 - \lambda_2 - n^2\right)e^{-\lambda_2 x} + \frac{\bar{a}_o\left(1 - n^2\right)}{P_2}$$

$$\left. + \sum_{m=1}^{M} \frac{\left\{\left[\left(\frac{m\pi}{x_1}\right)^3 + \frac{m\pi}{x_1}\left(1 - n^2\right)\right]\cos\frac{m\pi x}{x_1} - n^2\left(\frac{m\pi}{x_1}\right)^2 \sin\frac{m\pi x}{x_1}\right\} a_m}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\right) \cos n\phi \tag{S16}$$

and

$$N_{y\theta B} = \frac{nB_\theta\left(1 - \mu'_\theta\mu'_y\right)\cot\alpha}{y_1e^x}\left\{A_1\lambda_1e^{\lambda_1x} - A_2\lambda_1e^{-\lambda_1x} + A_3\lambda_2e^{\lambda_2x} - A_4\lambda_2e^{-\lambda_2x}\right.$$

$$\left. + \sum_{m=1}^{M}\frac{\left[\left(\frac{m\pi}{x_1}\right)^3\cos\frac{m\pi x}{x_1} - \left(\frac{m\pi}{x_1}\right)\sin\frac{m\pi x}{x_1}\right]}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}a_m\right\}\sin n\phi \tag{S17}$$

The constitutive equations are

$$\epsilon_{\theta B} = \frac{1}{B_\theta\left(1 - \mu'_\theta\mu'_y\right)}\left(N_{\theta B} - \mu'_\theta N_{yB}\right) \tag{S18a}$$

$$\epsilon_{yB} = \frac{1}{B_\theta\left(1 - \mu'_\theta\mu'_y\right)}\left(N_{yB} - \mu'_y N_{\theta B}\right) \tag{S18b}$$

$$\epsilon_{y\theta B} = \frac{1}{B_{y\theta}}N_{y\theta B} \tag{S18c}$$

and the linear strain-displacement relations are

$$\epsilon_{yB} = \frac{1}{y_1e^x}u_{B,x} \tag{S19a}$$

$$\epsilon_{\theta B} = \frac{1}{y_1e^x}\left(u_B - w_B\cot\alpha + v_{B,\phi}\right) \tag{S19b}$$

$$\epsilon_{y\theta B} = \frac{1}{y_1e^x}\left(v_{B,x} - v_B + u_{B,\phi}\right) \tag{S19c}$$

Substituting equations (S16), (S17), (S19a), and (S19b) into equation (S18b) and integrating give

$$u_B = \left\{\frac{B_\theta}{B_y}\cot\alpha\left[A_1\psi_1 e^{\lambda_1 x} + A_2\psi_2 e^{-\lambda_1 x} + A_3\psi_3 e^{\lambda_2 x} + A_4\psi_4 e^{-\lambda_2 x} - \frac{\bar{a}_o}{P_2}\left(n^2 - 1\right)x\right.\right.$$

$$+ \sum_{m=1}^{M}\frac{a_m}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\left(\left[\left(\frac{m\pi}{x_1}\right)^2 + 1 - n^2\right]\sin\frac{m\pi x}{x_1} - \left\{\mu_y'\left[\left(\frac{m\pi}{x_1}\right)^3 + \frac{m\pi}{x_1}\right]\right.\right.$$

$$\left.\left.\left.\left. - \frac{m\pi}{x_1}n^2\right\}\cos\frac{m\pi x}{x_1}\right)\right] + C_1\right\}\cos n\phi \qquad \text{(S20)}$$

where

$$\left.\begin{aligned}
\psi_1 &= \frac{1 + \lambda_1 - n^2}{\lambda_1} - \mu_y'\left(1 + \lambda_1\right)\\
\psi_2 &= \frac{\lambda_1 + n^2 - 1}{\lambda_1} - \mu_y'\left(1 - \lambda_1\right)\\
\psi_3 &= \frac{1 + \lambda_2 - n^2}{\lambda_2} - \mu_y'\left(1 + \lambda_2\right)\\
\psi_4 &= \frac{\lambda_2 + n^2 - 1}{\lambda_2} - \mu_y'\left(1 - \lambda_2\right)
\end{aligned}\right\} \qquad \text{(S21)}$$

Substituting equation (S17) and equation (S19c) into equation (S18c), utilizing equation (S20), and integrating give

$$v_B = n\cot\alpha\left[A_1\Lambda_1 e^{\lambda_1 x} + A_2\Lambda_2 e^{-\lambda_1 x} + A_3\Lambda_3 e^{\lambda_2 x} + A_4\Lambda_4 e^{-\lambda_2 x} + \frac{1 + x}{n^2 - 1}\bar{a}_o - \frac{C_1}{\cot\alpha} + \frac{C_2 e^x}{\cot\alpha}\right.$$

$$+ \sum_{m=1}^{M}\frac{a_m}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\left(\left\{\left(\frac{m\pi}{x_1}\right)^2\left[\frac{B_\theta}{B_{y\theta}}\left(1 - \mu_\theta'\mu_y'\right) - \mu_\theta'\right] + \frac{B_\theta}{B_y}\left(n^2 - 1\right)\right\}\sin\frac{m\pi x}{x_1}\right.$$

$$\left.\left. - \frac{m\pi}{x_1}\left(1 - \mu_y'\right)\frac{B_\theta}{B_y}\cos\frac{m\pi x}{x_1}\right)\right]\sin n\phi \qquad \text{(S22)}$$

where

$$\left.\begin{aligned}
\Lambda_1 &= \frac{\left(B_\theta/B_{y\theta}\right)\left(1 - \mu'_\theta\mu'_y\right)\lambda_1 + \left(B_\theta/B_y\right)\psi_1}{\lambda_1 - 1} \\
\Lambda_2 &= \frac{\left(B_\theta/B_{y\theta}\right)\left(1 - \mu'_\theta\mu'_y\right)\lambda_1 - \left(B_\theta/B_y\right)\psi_2}{\lambda_1 + 1} \\
\Lambda_3 &= \frac{\left(B_\theta/B_{y\theta}\right)\left(1 - \mu'_\theta\mu'_y\right)\lambda_2 + \left(B_\theta/B_y\right)\psi_3}{\lambda_2 - 1} \\
\Lambda_4 &= \frac{\left(B_\theta/B_{y\theta}\right)\left(1 - \mu'_\theta\mu'_y\right)\lambda_2 - \left(B_\theta/B_y\right)\psi_4}{\lambda_2 + 1}
\end{aligned}\right\} \tag{S23}$$

The constants of integration C_1 and C_2 are determined by substituting the expressions for u_B, v_B, w_B, $N_{\theta B}$, and N_{yB} into equation (S18a) and matching appropriate coefficients; this gives

$$\left.\begin{aligned}
C_1 &= \cot\alpha\left[\frac{\bar{a}_o}{P_2}\left(n^2\frac{B_\theta}{B_y} - \mu'_\theta\right) - \frac{a_o}{n^2 - 1}\right] \\
C_2 &= 0
\end{aligned}\right\} \tag{S24}$$

The constants of integration A_1 to A_4 are determined from the boundary condition for u_B and v_B, the first two of equations (B47). When the expressions for u_B, v_B, w_B, N_{yB}, and $N_{y\theta B}$ are substituted into equations (B47), the boundary conditions become

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & l_{34} \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1j} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{2j} \\ g_{31} & g_{32} & g_{33} & \cdots & g_{3j} \\ g_{41} & g_{42} & g_{43} & \cdots & g_{4j} \end{bmatrix} \begin{Bmatrix} a_o \\ \bar{a}_o \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_M \end{Bmatrix} \qquad (j = M + 2) \tag{S25}$$

where l_{ij} and g_{ij} are given in the next section.

Equation (S25) can be solved for A_1 to A_4 in the form

$$A_i = \Omega_i a_o + \Gamma_i \bar{a}_o + \sum_{m=1}^{M} \Phi_{im} a_m \qquad (i = 1, 2, 3, 4) \quad \text{(S26)}$$

With A_i known in terms of a_o, $\bar{a}_o$, and a_m, then F, u_B, and v_B can be expressed in terms of these coefficients as

$$F = \cot \alpha\, B_\theta \left(1 - \mu'_\theta \mu'_y\right) y_1 e^x \left\{ a_o\left(\Omega_1 e^{\lambda_1 x} + \Omega_2 e^{-\lambda_1 x} + \Omega_3 e^{\lambda_2 x} + \Omega_4 e^{-\lambda_2 x}\right) + \bar{a}_o\left(\Gamma_1 e^{\lambda_1 x} \right.\right.$$

$$\left. + \Gamma_2 e^{-\lambda_1 x} + \Gamma_3 e^{\lambda_2 x} + \Gamma_4 e^{-\lambda_2 x} + \frac{1}{P_2}\right) + \sum_{m=1}^{M} a_m \left[\Phi_{1m} e^{\lambda_1 x} + \Phi_{2m} e^{-\lambda_1 x} + \Phi_{3m} e^{\lambda_2 x}\right.$$

$$\left.\left. + \Phi_{4m} e^{-\lambda_2 x} + \frac{\left(\frac{m\pi}{x_1}\right)^2 \sin \frac{m\pi x}{x_1} + \frac{m\pi}{x_1} \cos \frac{m\pi x}{x_1}}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\right]\right\} \cos n\phi \qquad \text{(S27)}$$

$$u_B = \cot \alpha \left(B_\theta/B_y\right)\left\{ a_o\left[\Omega_1 \psi_1 e^{\lambda_1 x} + \Omega_2 \psi_2 e^{-\lambda_1 x} + \Omega_3 \psi_3 e^{\lambda_2 x} + \Omega_4 \psi_4 e^{-\lambda_2 x} + \frac{1}{\left(B_\theta/B_y\right)\left(1 - n^2\right)}\right]\right.$$

$$+ \bar{a}_o\left[\Gamma_1 \psi_1 e^{\lambda_1 x} + \Gamma_2 \psi_2 e^{-\lambda_1 x} + \Gamma_3 \psi_3 e^{\lambda_2 x} + \Gamma_4 \psi_4 e^{-\lambda_2 x} - \frac{x}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)} + \frac{n^2 - \mu'_y}{P_2}\right]$$

$$+ \sum_{m=1}^{M} a_m \left[\Phi_{1m} \psi_1 e^{\lambda_1 x} + \Phi_{2m} \psi_2 e^{-\lambda_1 x} + \Phi_{3m} \psi_3 e^{\lambda_2 x} + \Phi_{4m} \psi_4 e^{-\lambda_2 x}\right.$$

$$+ \frac{1}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\left(\left[\left(\frac{m\pi}{x_1}\right)^2 + 1 - n^2\right] \sin \frac{m\pi x}{x_1} - \left\{\mu'_y \left[\left(\frac{m\pi}{x_1}\right)^3 + \frac{m\pi}{x_1}\right]\right.\right.$$

$$\left.\left.\left.\left. - n^2 \frac{m\pi}{x_1}\right\} \cos \frac{m\pi x}{x_1}\right)\right]\right\} \cos n\phi \qquad \text{(S28)}$$

and

$$v_B = n \cot \alpha \left\{ a_o\left(\Lambda_1\Omega_1 e^{\lambda_1 x} + \Lambda_2\Omega_2 e^{-\lambda_1 x} + \Lambda_3\Omega_3 e^{\lambda_2 x} + \Lambda_4\Omega_4 e^{-\lambda_2 x} + \frac{1}{n^2 - 1}\right) + \bar{a}_o\left[\Lambda_1\Gamma_1 e^{\lambda_1 x}\right.\right.$$

$$+ \Lambda_2\Gamma_2 e^{-\lambda_1 x} + \Lambda_3\Gamma_3 e^{\lambda_2 x} + \Lambda_4\Gamma_4 e^{-\lambda_2 x} + \frac{x}{n^2 - 1} + \left.\frac{\mu_y' - 1}{\left(n^2 - 1\right)^2}\right] + \sum_{m=1}^{M} a_m\left[\Lambda_1\Phi_{1m} e^{\lambda_1 x}\right.$$

$$+ \Lambda_2\Phi_{2m} e^{-\lambda_1 x} + \Lambda_3\Phi_{3m} e^{\lambda_2 x} + \Lambda_4\Phi_{4m} e^{-\lambda_2 x} + \frac{1}{\left(\frac{m\pi}{x_1}\right)^4 - P_1\left(\frac{m\pi}{x_1}\right)^2 + P_2}\left(\left\{\frac{m\pi}{x_1}\left[\frac{B_{y\theta}}{B_y}\left(1\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.- \mu_\theta'\mu_y'\right) - \mu_\theta'\right] + \frac{B_\theta}{B_y}\left(n^2 - 1\right)\right\} \sin \frac{m\pi x}{x_1} - \frac{m\pi}{x_1}\frac{B_\theta}{B_y}\left(1 - \mu_y'\right) \cos \frac{m\pi x}{x_1}\right)\right]\right\} \sin n\phi \tag{S29}$$

For symmetric deformations the transformed compatibility equation is

$$F_{o,xx} - \frac{B_\theta}{B_y} F_o = \cot \alpha \, B_\theta\left(1 - \mu_\theta'\mu_y'\right)w^*_{B,x} \tag{S30}$$

and the solution is readily found to be

$$F_o = B_\theta\left(1 - \mu_\theta'\mu_y'\right) \cot \alpha \left[B_1 e^{\lambda_o x} + B_2 e^{-\lambda_o x} + \frac{\bar{a}_o}{B_\theta/B_y} + \sum_{m=1}^{M} \frac{\frac{m\pi}{x_1} \cos \frac{m\pi}{x_1}}{\left(m\pi/x_1\right)^2 + \left(B_\theta/B_y\right)} a_m\right] \tag{S31}$$

where

$$\lambda_o = \sqrt{B_\theta/B_y}$$

The displacement $v_{oB} = 0$ and u_{oB} is given by

$$
u_{oB} = \cot\alpha\left[B_1\left(\lambda_o - \mu'_\theta\right)e^{\lambda_o x} - B_2\left(\lambda_o + \mu'_\theta\right)e^{-\lambda_o x}\right.
$$

$$
\left. + \sum_{m=1}^{M} \frac{\left(B_\theta/B_y\right)\left(\sin\frac{m\pi x}{x_1} - \mu'_y\frac{m\pi}{x_1}\cos\frac{m\pi x}{x_1}\right)a_m}{\left(m\pi/x_1\right)^2 + \left(B_\theta/B_y\right)} + a_o + \bar{a}_o\left(x - \frac{\mu'_\theta}{B_\theta/B_y}\right)\right] \tag{S32}
$$

The boundary conditions become

$$
\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} = \begin{bmatrix} \bar{e}_{11} & \bar{e}_{12} & \bar{e}_{13} & \cdots & \bar{e}_{1j} \\ \bar{e}_{21} & \bar{e}_{22} & \bar{e}_{23} & \cdots & \bar{e}_{2j} \end{bmatrix}\begin{Bmatrix} a_o \\ \bar{a}_o \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_M \end{Bmatrix} \qquad (j = M + 2) \tag{S33}
$$

where h_{ij} and $\bar{e}_{ij}$ are given in the next section. Equation (S33) can be solved for B_1 and B_2 in the form

$$
B_i = \Omega_{oi} + \Gamma_{oi}\bar{a}_o + \sum_{m=1}^{M} \Phi_{oim}a_m \qquad (i = 1, 2) \tag{S34}
$$

With B_i known in terms of a_o, $\bar{a}_o$, and a_m, then F_o and u_{oB} can be expressed in terms of these coefficients as

$$
F_o = B_\theta\left(1 - \mu'_\theta\mu'_y\right)\cot\alpha\left\{a_o\left(\Omega_{o1}e^{\lambda_o x} + \Omega_{o2}e^{-\lambda_o x}\right) + \bar{a}_o\left(\Gamma_{o1}e^{\lambda_o x} + \Gamma_{o2}e^{-\lambda_o x} + \frac{1}{B_\theta/B_y}\right)\right.
$$

$$
\left. + \sum_{m=1}^{M} a_m\left[\Phi_{o1m}e^{\lambda_o x} + \Phi_{o2m}e^{-\lambda_o x} + \frac{\left(m\pi/x_1\right)\cos\left(m\pi x/x_1\right)}{\left(m\pi/x_1\right)^2 + \left(B_\theta/B_y\right)}\right]\right\} \tag{S35}
$$

$$
\begin{aligned}
u_{oB} = \cot\alpha \Bigg\{ & a_o\left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)e^{\lambda_o x} - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)e^{-\lambda_o x} + 1\right] \\
& + \bar{a}_o\left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)e^{\lambda_o x} - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)e^{-\lambda_o x} + x - \frac{\mu'_\theta}{B_\theta/B_y}\right] \\
& + \sum_{m=1}^{M} a_m\left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)e^{\lambda_o x} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)e^{-\lambda_o x}\right. \\
& \left.\left. + \frac{B_\theta/B_y}{\left(m\pi/x_1\right)^2 + \left(B_\theta/B_y\right)}\left(\sin\frac{m\pi x}{x_1} - \frac{B_y}{B_\theta}\mu'_\theta\,\frac{m\pi}{x_1}\cos\frac{m\pi x}{x_1}\right)\right]\right\}
\end{aligned}
\tag{S36}
$$

EXPRESSIONS FOR l_{ij}, g_{ij}, h_{ij}, AND $\bar{e}_{ij}$

The expressions for l_{ij} are

$$l_{11} = \left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 + \lambda_1 - n^2\right) - \cot\alpha \frac{B_\theta}{B_y}\frac{Q_{11,1}}{Z}\psi_1 - n\cot\alpha \frac{Q_{12,1}}{Z}\Lambda_1$$

$$l_{12} = \left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 - \lambda_1 - n^2\right) - \cot\alpha \frac{B_\theta}{B_y}\frac{Q_{11,1}}{Z}\psi_2 - n\cot\alpha \frac{Q_{12,1}}{Z}\Lambda_2$$

$$l_{13} = \left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 + \lambda_2 - n^2\right) - \cot\alpha \frac{B_\theta}{B_y}\frac{Q_{11,1}}{Z}\psi_3 - n\cot\alpha \frac{Q_{12,1}}{Z}\Lambda_3$$

$$l_{14} = \left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 - \lambda_2 - n^2\right) - \cot\alpha \frac{B_\theta}{B_y}\frac{Q_{11,1}}{Z}\psi_4 - n\cot\alpha \frac{Q_{12,1}}{Z}\Lambda_4$$

$$l_{21} = \left(\frac{R_2}{R_1}\right)^{\lambda_1}\left[\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 + \lambda_1 - n^2\right) + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{11,2}}{Z}\psi_1 + n\frac{Q_{12,2}}{Z}\Lambda_1\right)\right]$$

$$l_{22} = \left(\frac{R_2}{R_1}\right)^{-\lambda_1}\left[\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 - \lambda_1 - n^2\right) + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{11,2}}{Z}\psi_2 + n\frac{Q_{12,2}}{Z}\Lambda_2\right)\right]$$

$$l_{23} = \left(\frac{R_2}{R_1}\right)^{\lambda_2}\left[\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 + \lambda_2 - n^2\right) + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{11,2}}{Z}\psi_3 + n\frac{Q_{12,2}}{Z}\Lambda_3\right)\right]$$

$$l_{24} = \left(\frac{R_2}{R_1}\right)^{-\lambda_2}\left[\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left(1 - \lambda_2 - n^2\right) + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{11,2}}{Z}\psi_4 + n\frac{Q_{12,2}}{Z}\Lambda_4\right)\right]$$

$$l_{31} = -n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_1 - \cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,1}}{Z}\psi_1 + n\frac{Q_{22,1}}{Z}\Lambda_1\right)$$

$$l_{32} = -n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_1 - \cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,1}}{Z}\psi_2 + n\frac{Q_{22,1}}{Z}\Lambda_2\right)$$

$$l_{33} = -n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_2 - \cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,1}}{Z}\psi_3 + n\frac{Q_{22,1}}{Z}\Lambda_3\right)$$

$$l_{34} = -n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_2 - \cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,1}}{Z}\psi_4 + n\frac{Q_{22,1}}{Z}\Lambda_4\right)$$

$$l_{41} = \left(\frac{R_2}{R_1}\right)^{\lambda_1}\left[n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_1 + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,2}}{Z}\psi_1 + n\frac{Q_{22,2}}{Z}\Lambda_1\right)\right]$$

$$l_{42} = \left(\frac{R_2}{R_1}\right)^{-\lambda_1}\left[-n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_1 + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,2}}{Z}\psi_2 + n\frac{Q_{22,2}}{Z}\Lambda_2\right)\right]$$

$$l_{43} = \left(\frac{R_2}{R_1}\right)^{\lambda_2}\left[n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_2 + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{21,2}}{Z}\psi_3 + n\frac{Q_{22,2}}{Z}\Lambda_3\right)\right]$$

$$l_{44} = \left(\frac{R_2}{R_1}\right)^{-\lambda_2}\left[-n\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\lambda_2 + \frac{R_2}{R_1}\cot\alpha\left(\frac{B_\theta}{B_y}\frac{Q_{12,1}}{Z}\psi_4 + n\frac{Q_{22,2}}{Z}\Lambda_4\right)\right]$$

The expressions for g_{ij} are

$$g_{11} = \frac{\cot\alpha}{n^2 - 1}\left(n\frac{Q_{12,1}}{Z} - \frac{Q_{11,1}}{Z}\right) + \frac{Q_{13,1}}{Z}$$

$$g_{12} = \frac{n^2 - 1}{P_2}\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right) + \cot\alpha\frac{Q_{11,1}}{Z}\left[\frac{n^2\left(B_\theta/B_y\right) - \mu_\theta'}{P_2}\right] + n\cot\alpha\frac{Q_{12,1}}{Z}\left[\frac{1}{n^2 - 1}\right.$$

$$\left. - \frac{n^2\left(B_\theta/B_y\right) - \mu_\theta'}{P_2}\right] + \frac{Q_{14,1}}{Z}\tan\alpha$$

$$g_{1i} = \frac{-m\pi/x_1}{\left(m\pi/x_1\right)^4 - P_1\left(m\pi/x_1\right)^2 + P_2}\left(\left(1 - \frac{\mu_\theta'^2}{B_\theta/B_y}\right)\left[\left(\frac{m\pi}{x_1}\right)^2 + 1 - n^2\right] + \cot\alpha\frac{Q_{11,1}}{Z}\left\{\mu_\theta'\left[\left(\frac{m\pi}{x_1}\right)^2 + 1\right]\right.\right.$$

$$\left.\left. - \frac{B_\theta}{B_y}n^2\right\} + n\cot\alpha\frac{Q_{12,1}}{Z}\left(\frac{B_\theta}{B_y} - \mu_\theta'\right)\right) + \frac{Q_{14,1}}{Z}\frac{m\pi}{x_1}\tan\alpha \qquad (i > 2;\quad m = i - 2)$$

$$g_{21} = \frac{R_2}{R_1}\left[\frac{\cot\alpha}{n^2 - 1}\left(\frac{Q_{11,2}}{Z} - n\,\frac{Q_{12,2}}{Z}\right) - \frac{Q_{13,2}}{Z}\right]$$

$$g_{22} = \frac{n^2 - 1}{P_2}\left(1 - \frac{\mu_\theta'^2}{B\theta/By}\right) + \frac{R_2}{R_1}\,\frac{\cot\alpha}{\left(n^2 - 1\right)^2}\,\frac{Q_{11,2}}{Z}\left[\left(n^2 - 1\right)\ln\frac{R_2}{R_1} - n^2 + \frac{\mu_\theta'}{B\theta/By}\right]$$

$$- \frac{R_2}{R_1}\,n\cot\alpha\,\frac{Q_{12,2}}{Z}\left[\frac{1 + \ln R_2/R_1}{n^2 - 1} + \frac{n^2\left(B_\theta/B_y\right) - \mu_\theta'}{P_2}\right] - \frac{R_2}{R_1}\,\frac{Q_{13,2}}{Z}\ln\frac{R_2}{R_1} - \frac{Q_{14,2}}{Z}\tan\alpha$$

$$g_{2i} = \frac{(-1)^m\,m\pi/x_1}{\left(m\pi/x_1\right)^4 - P_1\left(m\pi/x_1\right)^2 + P_2}\left(-\left(1 - \frac{\mu_\theta'^2}{B\theta/By}\right)\left[\left(\frac{m\pi}{x_1}\right)^2 + 1 - n^2\right] + \frac{R_2}{R_1}\cot\alpha\,\frac{Q_{11,2}}{Z}\right.$$

$$\left.\times\left\{\mu_\theta'\left[\left(\frac{m\pi}{x_1}\right)^2 + 1\right] - \frac{B_\theta}{By}\,n^2\right\} + \frac{R_2}{R_1}\,n\cot\alpha\,\frac{Q_{12,2}}{Z}\left(\frac{B_\theta}{By} - \mu_\theta'\right)\right)$$

$$- \frac{Q_{14,2}}{Z}\tan\alpha\,(-1)^m\,\frac{m\pi}{x_1} \qquad (i > 2;\quad m = i - 2)$$

$$g_{31} = \frac{\cot\alpha}{n^2 - 1}\left(n\,\frac{Q_{22,1}}{Z} - \frac{Q_{21,1}}{Z}\right) + \frac{Q_{23,1}}{Z}$$

$$g_{32} = \frac{\cot\alpha}{\left(n^2 - 1\right)^2}\,\frac{Q_{21,1}}{Z}\left(n^2 - \frac{\mu_\theta'}{B\theta/By}\right) + n\cot\alpha\,\frac{Q_{22,1}}{Z}\left[\frac{1}{n^2 - 1} - \frac{\left(n^2 - \frac{\mu_\theta'}{B\theta/By}\right)}{\left(n^2 - 1\right)^2}\right] + \frac{Q_{24,1}}{Z}\tan\alpha$$

$$g_{3i} = \frac{-m\pi/x_1}{\left(m\pi/x_1\right)^4 - P_1\left(m\pi/x_1\right)^2 + P_2}\left(-n\left(1 - \frac{\mu_\theta'^2}{B\theta/By}\right)\left(\frac{m\pi}{x_1}\right)^2 + \cot\alpha\,\frac{Q_{21,1}}{Z}\left\{\mu_\theta'\left[\left(\frac{m\pi}{x_1}\right)^2 + 1\right]\right.\right.$$

$$\left.\left.- \frac{B_\theta}{B_y}\,n^2\right\} + n\cot\alpha\,\frac{Q_{22,1}}{Z}\left(\frac{B_\theta}{B_y} - \mu_\theta'\right)\right) + \frac{Q_{24,1}}{Z}\tan\alpha\,\frac{m\pi}{x_1} \qquad (i > 2;\quad m = i - 2)$$

$$g_{41} = \frac{R_2}{R_1}\left[\frac{\cot\alpha}{n^2 - 1}\left(\frac{Q_{21,2}}{Z} - n\frac{Q_{22,2}}{Z}\right) - \frac{Q_{23,2}}{Z}\right]$$

$$g_{42} = \frac{R_2}{R_1}\frac{\cot\alpha}{\left(n^2 - 1\right)^2}\frac{Q_{21,2}}{Z}\left[\left(n^2 - 1\right)\ln\frac{R_2}{R_1} - \left(n^2 - \frac{\mu'_\theta}{B_\theta/B_y}\right)\right] - \frac{R_2}{R_1} n\cot\alpha\frac{Q_{22,2}}{Z}\left[\frac{1 + \ln R_2/R_1}{n^2 - 1}\right.$$

$$\left. - \frac{n^2\left(B_\theta/B_y\right) - \mu'_\theta}{P_2}\right] - \frac{R_2}{R_1}\frac{Q_{23,2}}{Z}\ln\frac{R_2}{R_1} - \frac{Q_{24,2}}{Z}\tan\alpha$$

$$g_{4i} = \frac{(-1)^m m\pi/x_1}{\left(m\pi/x_1\right)^4 - P_1\left(m\pi/x_1\right)^2 + P_2}\left(n\left(1 - \frac{\mu'^2_\theta}{B_\theta/B_y}\right)\left(\frac{m\pi}{x_1}\right)^2 + \frac{R_2}{R_1}\cot\alpha\frac{Q_{21,2}}{Z}\left\{\mu'_\theta\left[\left(\frac{m\pi}{x_1}\right)^2 + 1\right]\right.\right.$$

$$\left.\left. - \frac{B_\theta}{B_y}n^2\right\} + \frac{R_2}{R_1}n\cot\alpha\frac{Q_{22,2}}{Z}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)\right) - \frac{Q_{24,2}}{Z}\tan\alpha\,(-1)^m\frac{m\pi}{x_1}$$

$$(i > 2;\quad m = i - 2)$$

The expressions for h_{ij} are

$$h_{11} = 1 - \frac{\mu'^2_\theta}{B_\theta/B_y} - \frac{Q_{11,1}}{Z}\cot\alpha\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)$$

$$h_{12} = 1 - \frac{\mu'^2_\theta}{B_\theta/B_y} + \frac{Q_{11,1}}{Z}\cot\alpha\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)$$

$$h_{21} = \left(\frac{R_2}{R_1}\right)^{\lambda_o}\left[1 - \frac{\mu'^2_\theta}{B_\theta/B_y} + \frac{R_2}{R_1}\frac{Q_{11,2}}{Z}\cot\alpha\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\right]$$

$$h_{22} = \left(\frac{R_2}{R_1}\right)^{-\lambda_o}\left[1 - \frac{\mu'^2_\theta}{B_\theta/B_y} - \frac{R_2}{R_1}\frac{Q_{11,2}}{Z}\cot\alpha\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\right]$$

The expressions for $\bar{e}_{ij}$ are

$$\bar{e}_{11} = \frac{Q_{11,1}}{Z} \cot \alpha + \frac{Q_{13,1}}{Z}$$

$$\bar{e}_{12} = \frac{1 - \mu'_\theta \mu'_y}{B_\theta/B_y} - \frac{Q_{11,1}}{Z} \cot \alpha \frac{\mu'_\theta}{B_\theta/B_y} + \frac{Q_{14,1}}{Z} \tan \alpha$$

$$\bar{e}_{1j} = \frac{-m\pi/x_1}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \left(1 - \mu'_\theta \mu'_y + \frac{Q_{11,1}}{Z} \cot \alpha \, \mu'_\theta\right) + \frac{m\pi}{x_1} \frac{Q_{14,1}}{Z} \tan \alpha \qquad (j > 2; \quad m = j - 2)$$

$$\bar{e}_{21} = -\frac{R_2}{R_1}\left(\frac{Q_{11,2}}{Z} \cot \alpha + \frac{Q_{13,2}}{Z}\right)$$

$$\bar{e}_{22} = -\left[\frac{1 - \mu'_\theta \mu'_y}{B_\theta/B_y} + \frac{R_2}{R_1} \cot \alpha \frac{Q_{11,2}}{Z}\left(\ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y}\right) + \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \frac{Q_{13,2}}{Z} + \frac{Q_{14,2}}{Z} \tan \alpha\right]$$

$$\bar{e}_{2j} = (-1)^m \frac{m\pi}{x_1} \frac{\left(R_2/R_1\right) \cot \alpha \, \mu'_\theta Q_{11,2} - \left(1 - \mu'_\theta \mu'_y\right)}{\left(m\pi/x_1\right)^2 + \left(B_\theta/B_y\right)} - \frac{Q_{14,2}}{Z} \tan \alpha \qquad (j > 2; \quad m = j - 2)$$

where

$$Z = \left(\frac{R_1}{\cos \alpha}\right)^2 \frac{B_\theta}{D_y}$$

$$Q_{ij} = S_{ij} - \frac{C_\omega M_{ij}}{\cos^4 \alpha}$$

EXPRESSIONS FOR b_{ij}, c_{ij}, d_{ij}, e_{ij}, AND f_{ij}

The expressions for b_{ij} are

$$b_{11} = n^2\left[1 - \left(\frac{R_2}{R_1}\right)^{-2}\right]\left(\frac{n^2}{2}\frac{D_\theta}{D_y} + \frac{D_{y\theta}}{D_y}\right) - Z\left(1 - \mu'_\theta\mu'_y\right)t_1 \cot^4 \alpha + \gamma_1$$

$$b_{12} = \frac{n^2}{4}\left[1 - \left(\frac{R_2}{R_1}\right)^{-2}\right]\left[\frac{D_\theta}{D_y}\left(n^2 - 2\right) - 2\left(\frac{D_{y\theta}}{D_y} - \mu_\theta\right)\right] - n^2\left(\frac{n^2}{2}\frac{D_\theta}{D_y} + \frac{D_{y\theta}}{D_y}\right)\left(\frac{R_2}{R_1}\right)^{-2} \ln \frac{R_2}{R_1}$$

$$- Z\left(1 - \mu'_\theta\mu'_y\right) t_2 \cot^4 \alpha + \gamma_2$$

$$b_{21} = b_{12}$$

with t_2 and γ_2 replaced by t_4 and γ_4.

$$b_{1j} = n^2 \frac{m\pi}{\ln R_2/R_1} \frac{1 - (-1)^m\left(R_2/R_1\right)^{-2}}{4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2}\left\{\mu_\theta\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 2\right] + \frac{D_\theta}{D_y}\left(n^2 - 2\right) - 2\frac{D_{y\theta}}{D_y}\right\}$$

$$- Z\left(1 - \mu'_\theta\mu'_y\right) t_{3m} \cot^4 \alpha + \gamma_{3m} \qquad (j > 2;\quad m = j - 2)$$

$$b_{22} = \frac{1}{4}\left[1 - \left(\frac{R_2}{R_1}\right)^{-2}\right]\left[\frac{D_\theta}{D_y}\left(n^4 - 2n^2 + 2\right) + 2n^2\frac{D_{y\theta}}{D_y} + 2\mu_\theta\left(n^2 - 2\right) + 2\right] - n^2\left(\frac{R_2}{R_1}\right)^{-2} \ln \frac{R_2}{R_1}$$

$$\times\left[\frac{D_\theta}{D_y}\left(\frac{n^2}{2} - 1\right) + \mu_\theta - \frac{D_{y\theta}}{D_y}\right] - \frac{n^2}{2}\left(\frac{R_2}{R_1}\right)^{-2}\left(\ln \frac{R_2}{R_1}\right)^2\left(n^2\frac{D_\theta}{D_y} + 2\frac{D_{y\theta}}{D_y}\right)$$

$$- Z\left(1 - \mu'_\theta\mu'_y\right) t_5 \cot^4 \alpha + \gamma_5$$

$$b_{2j} = \frac{m\pi}{\ln R_2/R_1}\left[\frac{n^2(-1)^m\left(R_2/R_1\right)^{-2} \ln R_2/R_1}{4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2}\left\{\frac{D_\theta}{D_y}\left(2 - n^2\right) + 2\frac{D_{y\theta}}{D_y} - \mu_\theta\left[2 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]\right\}\right.$$

(Equation continued on next page)

$$+\frac{1-(-1)^m\left(R_2/R_1\right)^{-2}}{\left[4+\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2}\left(\mu_\theta\left\{n^2\left[8+4\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]-\left[4+\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2\right\}+\frac{D_\theta}{D_y}\left[2\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right.\right.$$

$$\left.\left.+8\left(1-n^2\right)+4n^4\right]+n^2\frac{D_{y\theta}}{D_y}\left[8+4\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]-2\left(\frac{m\pi}{\ln R_2/R_1}\right)^2-8+\left[4+\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2\right)\Bigg]$$

$$-Z\left(1-\mu'_\theta\mu'_y\right)t_{6m}\cot^4\alpha+\gamma_{6m} \qquad (j>2;\quad m=j-2)$$

$$b_{i1}=b_{1j} \qquad (i>2;\quad p=i-2)$$

with t_{3m} and γ_{3m} replaced by t_{7p} and γ_{7p} and m replaced by p.

$$b_{i2}=b_{2j} \qquad (i>2;\quad p=i-2)$$

with t_{6m} and γ_{6m} replaced by t_{8p} and γ_{8p} and m replaced by p.

$$b_{jj}=\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\left(\frac{1-\left(R_2/R_1\right)^{-2}}{4\left[1+\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]}\left\{\frac{D_\theta}{D_y}\left[\left(\frac{p\pi}{\ln R_2/R_1}\right)^2+2+n^2\left(n^2-2\right)\right]\right.\right.$$

$$\left.+2n^2\frac{D_{y\theta}}{D_y}\left[1+\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]+2n^2\mu_\theta\left[1+\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]+\left(\frac{p\pi}{\ln R_2/R_1}\right)^4-\left(\frac{p\pi}{\ln R_2/R_1}\right)^2-2\right\}$$

$$\left.+\left[1-\left(\frac{R_2}{R_1}\right)^{-2}\right]\left(1-\mu_\theta\right)\right)-Z\left(1-\mu'_\theta\mu'_y\right)t_{9p}\cot^4\alpha+\gamma_{9p} \qquad (j>2;\quad p=j-2)$$

$$b_{ij} = \frac{m\pi}{\ln R_2/R_1} \frac{p\pi}{\ln R_2/R_1} \left[\left[1 - (-1)^{m+p}\left(\frac{R_2}{R_1}\right)^{-2}\right]\left(\frac{1}{\left\{4 + \left[\frac{(m - p)\pi}{\ln R_2/R_1}\right]^2\right\}\left\{4 + \left[\frac{(m + p)\pi}{\ln R_2/R_1}\right]^2\right\}}\right.\right.$$

$$\times \left\{\left(\frac{D_\theta}{D_y} - 1\right)\left[8 + 2\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 2\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right] + 4n^2 \frac{D_\theta}{D_y}\left(n^2 - 2\right) + 4n^2\left(\frac{D_{y\theta}}{D_y} + \mu_\theta\right)\right.$$

$$\left.\left.\left.\times \left[2 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + \left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right] + 4\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right\} + 1 - \mu_\theta\right)\right]$$

$$- Z\left(1 - \mu'_\theta \mu'_y\right) t_{9mp} \cot^4 \alpha + \gamma_{9mp} \qquad (i,j > 2; \quad m = j - 2; \quad p = i - 2)$$

The expressions for c_{ij} are

$$c_{11} = c_{12} = c_{21} = c_{1j} = c_{i1} = 0$$

$$c_{22} = -\left[1 - \left(\frac{R_2}{R_1}\right)^{-1}\right]$$

$$c_{2j} = -\frac{m\pi}{\ln R_2/R_1}\left[\frac{1 - (-1)^m \left(R_2/R_1\right)^{-1}}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2}\right] = c_{i2} \qquad (j > 2; \quad m = j - 2)$$

$$c_{jj} = -\frac{\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\left[1 + 2\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]\left[1 - \left(R_2/R_1\right)^{-1}\right]}{1 + 4\left(\frac{p\pi}{\ln R_2/R_1}\right)^2} \qquad (j > 2; \quad p = j - 2)$$

$$c_{ij} = -\frac{m\pi}{\ln R_2/R_1}\,\frac{p\pi}{\ln R_2/R_1}\,\frac{1 - (-1)^{m+p}\left(R_2/R_1\right)^{-1}\left[1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + \left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]}{\left\{1 + \left[\frac{(m - p)\pi}{\ln R_2/R_1}\right]^2\right\}\left\{1 + \left[\frac{(m + p)\pi}{\ln R_2/R_1}\right]^2\right\}}$$

$(i,j > 2; \quad m = j - 2; \quad p = i - 2)$

The expressions for d_{ij} are

$$d_{11} = -n^2\left(\frac{R_2}{R_1} - 1\right)$$

$$d_{12} = d_{21} = -n^2\left[\frac{R_2}{R_1}\left(\ln\frac{R_2}{R_1} - 1\right) + 1\right]$$

$$d_{1j} = -n^2\,\frac{m\pi}{\ln R_2/R_1}\left[\frac{1 - (-1)^m\left(R_2/R_1\right)}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2}\right] = d_{i1} \qquad (j > 2; \quad m = j - 2)$$

with m replaced by p.

$$d_{22} = \frac{1}{2}\left(1 - \frac{R_2}{R_1}\right) - n^2\left[2\left(\frac{R_2}{R_1} - 1\right) + \frac{R_2}{R_1}\ln\frac{R_2}{R_1}\left(\ln\frac{R_2}{R_1} - 2\right)\right]$$

$$d_{2j} = \frac{m\pi}{\ln R_2/R_1}\left\{\frac{n^2(-1)^m\left(R_2/R_1\right)\ln R_2/R_1 + \frac{1}{2}\left[1 - (-1)^m\left(R_2/R_1\right)\right]}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2}\right.$$

$$\left. + \frac{2n^2\left[1 - (-1)^m\left(R_2/R_1\right)\right]}{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2}\right\} = d_{i2} \qquad (j > 2; \quad m = j - 2)$$

with m replaced by p.

$$d_{jj} = \frac{1}{2}\left(\frac{p\pi}{\ln R_2/R_1}\right)^2 \left\{ \frac{\left[1 + 2\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\right]\left[1 - \left(R_2/R_1\right)\right] + 4n^2\left[1 - \left(R_2/R_1\right)\right]}{1 + 4\left(\frac{p\pi}{\ln R_2/R_1}\right)^2} \right\}$$

$(j > 2; \quad p = j - 2)$

$$d_{ij} = \frac{1}{2}\,\frac{m\pi}{\ln R_2/R_1}\,\frac{p\pi}{\ln R_2/R_1}\,\frac{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + \left(\frac{p\pi}{\ln R_2/R_1}\right)^2 + 4n^2\right]\left[1 - (-1)^{m+p}\left(R_2/R_1\right)\right]}{\left\{1 + \left[\frac{(m - p)\pi}{\ln R_2/R_1}\right]^2\right\}\left\{1 + \left[\frac{(m + p)\pi}{\ln R_2/R_1}\right]^2\right\}}$$

$(i,j > 2; \quad m = i - 2; \quad p = j - 2)$

The expressions for e_{ij} are

$$e_{11} = \frac{1}{2}\left[\left(\frac{R_2}{R_1}\right)^2 - 1\right] + \beta_1$$

$$e_{12} = \frac{1}{4}\left[\left(\frac{R_2}{R_1}\right)^2\left(2 \ln \frac{R_2}{R_1} - 1\right) + 1\right] + \beta_2$$

$$e_{1j} = \frac{m\pi}{\ln R_2/R_1}\,\frac{\left[1 - (-1)^m\left(R_2/R_1\right)^2\right]}{4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2} + \beta_{3m} \qquad (j > 2; \quad m = j - 2)$$

$$e_{21} = e_{12}$$

with β_2 replaced by β_4.

$$e_{22} = \frac{1}{4}\left\{\left(\frac{R_2}{R_1}\right)^2\left[2\left(\ln \frac{R_2}{R_1}\right)^2 - 2 \ln \frac{R_2}{R_1} + 1\right] + 1\right\} + \beta_5$$

$$e_{2j} = \frac{m\pi}{\ln R_2/R_1}\left\{- \frac{(-1)^m\left(R_2/R_1\right)^2 \ln R_2/R_1}{4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2} + \frac{4\left[(-1)^m\left(R_2/R_1\right)^2 - 1\right]}{\left[4 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2}\right\} + \beta_{6m}$$

$(j > 2; \quad m = j - 2)$

$e_{i1} = e_{1j}$ $\qquad (i > 2;\quad p = i - 2)$

with β_{3m} replaced by β_{7p} and m replaced by p.

$e_{i2} = e_{2j}$ $\qquad (i > 2;\quad p = i - 2)$

with β_{6m} replaced by β_{8m} and m replaced by p.

$$e_{ij} = 4\,\frac{p\pi}{\ln R_2/R_1}\,\frac{m\pi}{\ln R_2/R_1}\,\frac{(-1)^{p+m}\left(R_2/R_1\right)^2 - 1}{\left\{4 + \left[(p + m)\,\frac{\pi}{\ln R_2/R_1}\right]^2\right\}\left\{4 + \left[(p - m)\,\frac{\pi}{\ln R_2/R_1}\right]^2\right\}} + \beta_{9mp}$$

$(i,j > 2;\quad m = i - 2;\quad p = j - 2)$

The expressions for f_{ij} are

$$f_{11} = f_{21} = f_{i1} = 0$$

$$f_{12} = \frac{R_2}{R_1} - 1$$

$$f_{1j} = -\,\frac{m\pi}{\ln R_2/R_1}\,\frac{1 - (-1)^m\left(R_2/R_1\right)}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2} \qquad (j > 2;\quad m = j - 2)$$

$$f_{22} = \frac{R_2}{R_1}\left(\ln\frac{R_2}{R_1} - 1\right) + 1$$

$$f_{2j} = \frac{m\pi}{x_1}\left\{\frac{(-1)^m\left(R_2/R_1\right)\ln\left(R_2/R_1\right)}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2} + \frac{\left[1 - \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]\left[1 - (-1)^m\left(R_2/R_1\right)\right]}{\left[1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]^2}\right\}$$

$(j > 2;\quad m = j - 2)$

$$f_{i2} = \frac{m\pi}{\ln R_2/R_1}\,\frac{1 - (-1)^m\left(R_2/R_1\right)}{1 + \left(\frac{m\pi}{\ln R_2/R_1}\right)^2} \qquad (i > 2;\quad m = i - 2)$$

$$f_{ii} = -\frac{\left(\frac{m\pi}{\ln R_2/R_1}\right)^2\left(\frac{R_2}{R_1} - 1\right)}{1 + 4\left(\frac{m\pi}{\ln R_2/R_1}\right)^2} \qquad (i > 2; \quad m = i - 2)$$

$$f_{ij} = \frac{\frac{m\pi}{\ln R_2/R_1}\,\frac{p\pi}{\ln R_2/R_1}\left[1 - (-1)^{p+m}\left(R_2/R_1\right)\right]\left[1 + \left(\frac{p\pi}{\ln R_2/R_1}\right)^2 - \left(\frac{m\pi}{\ln R_2/R_1}\right)^2\right]}{\left\{1 + \left[\frac{(p + m)\pi}{\ln R_2/R_1}\right]^2\right\}\left\{1 + \left[\frac{(p - m)\pi}{\ln R_2/R_1}\right]^2\right\}}$$

$$(i,j > 2; \quad m = i - 2; \quad p = j - 2)$$

The quantities t are defined as

$$t_1 = \left(\lambda_1 + 1\right)\Omega_1\left[\left(\frac{R_2}{R_1}\right)^{\lambda_1} - 1\right] + \left(\lambda_1 - 1\right)\Omega_2\left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_1}\right] + \left(\lambda_2 + 1\right)\Omega_3\left[\left(\frac{R_2}{R_1}\right)^{\lambda_2} - 1\right]$$

$$+ \left(\lambda_2 - 1\right)\Omega_4\left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right]$$

$$t_2 = t_1$$

with Ω_i replaced by Γ_i.

$$t_3 = t_1$$

with Ω_i replaced by Φ_{im} and the additional term

$$-\frac{\frac{m\pi}{\ln R_2/R_1}\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right]\left[1 - (-1)^m\right]}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}$$

$$t_4 = \frac{\lambda_1 + 1}{\lambda_1}\,\Omega_1\left[1 + \left(\frac{R_2}{R_1}\right)^{\lambda_1}\left(\lambda_1 \ln\frac{R_2}{R_1} - 1\right)\right] + \frac{\lambda_1 - 1}{\lambda_1}\,\Omega_2\left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_1}\left(\lambda_1 \ln\frac{R_2}{R_1} + 1\right)\right]$$

(Equation continued on next page)

$$+\frac{\lambda_2 + 1}{\lambda_2}\Omega_3\left[1 + \left(\frac{R_2}{R_1}\right)^{\lambda_2}\left(\lambda_2 \ln \frac{R_2}{R_1} - 1\right)\right] + \frac{\lambda_2 - 1}{\lambda_2}\Omega_4\left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_2}\left(\lambda_2 \ln \frac{R_2}{R_1} + 1\right)\right]$$

$t_5 = t_4$

with Ω_i replaced by Γ_i.

$t_{6m} = t_4$

with Ω_i replaced by Φ_{im} and the additional term

$$+\frac{\dfrac{m\pi}{\ln R_2/R_1}\left[\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right](-1)^m \ln R_2/R_1}{\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}$$

$$t_{7p} = \frac{p\pi}{\ln R_2/R_1}\left\{\frac{\lambda_1(\lambda_1 + 1)\,\Omega_1\left[1 - (-1)^p\left(R_2/R_1\right)^{\lambda_1}\right]}{\lambda_1^2 + \left(\dfrac{p\pi}{\ln R_2/R_1}\right)^2} + \frac{\lambda_1(\lambda_1 - 1)\,\Omega_2\left[1 - (-1)^p\left(R_2/R_1\right)^{-\lambda_1}\right]}{\lambda_1^2 + \left(\dfrac{p\pi}{\ln R_2/R_1}\right)^2}\right.$$

$$\left.+\frac{\lambda_2(\lambda_2 + 1)\,\Omega_3\left[1 - (-1)^p\left(R_2/R_1\right)^{\lambda_2}\right]}{\lambda_2^2 + \left(\dfrac{p\pi}{\ln R_2/R_1}\right)^2} + \frac{\lambda_2(\lambda_2 - 1)\,\Omega_4\left[1 - (-1)^p\left(R_2/R_1\right)^{-\lambda_2}\right]}{\lambda_2^2 + \left(\dfrac{p\pi}{\ln R_2/R_1}\right)^2}\right\}$$

$t_{8p} = t_{7p}$

with Ω_i replaced by Γ_i.

$t_{9mp} = t_{7p}$

with Ω_i replaced by Φ_{im}.

$t_{9p} = t_{7p}$

with Ω_i replaced by Φ_{ip} and the additional term

$$
- \frac{\frac{1}{2}\left(\frac{p\pi}{\ln R_2/R_1}\right)^2\left[\left(\frac{p\pi}{\ln R_2/R_1}\right)^2 + 1\right] \ln R_2/R_1}{\left(\frac{p\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{p\pi}{\ln R_2/R_1}\right)^2 + P_2}
$$

The quantities γ are defined as

$$
\begin{aligned}
\gamma_1 = {} & \cot^3 \alpha\left(S_{33,1} + \frac{R_2}{R_1} S_{33,2}\right) + \cot^4 \alpha \frac{B_\theta}{B_y}\left\{S_{31,1}\left[\Omega_1\psi_1 + \Omega_2\psi_2 + \Omega_3\psi_3 + \Omega_4\psi_4\right.\right. \\
& \left. - \frac{1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)}\right] + S_{31,2} \frac{R_2}{R_1}\left[\Omega_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Omega_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Omega_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}\right. \\
& \left.\left. + \Omega_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)}\right]\right\} + n \cot^4 \alpha\left\{S_{32,1}\left(\Lambda_1\Omega_1 + \Lambda_2\Omega_2 + \Lambda_3\Omega_3\right.\right. \\
& \left. + \Lambda_4\Omega_4 + \frac{1}{n^2 - 1}\right) + S_{32,2} \frac{R_2}{R_1}\left[\Lambda_1\Omega_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Omega_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Omega_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}\right. \\
& \left.\left. + \Lambda_4\Omega_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{1}{n^2 - 1}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2 = {} & \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^3 \alpha \; S_{33,2} + \frac{B_\theta}{B_y} \cot^4 \alpha\left\{S_{31,1}\left(\Gamma_1\psi_1 + \Gamma_2\psi_2 + \Gamma_3\psi_3 + \Gamma_4\psi_4 + \frac{n^2 - \mu_y'}{P_2}\right)\right. \\
& + S_{31,2} \frac{R_2}{R_1}\left[\Gamma_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Gamma_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Gamma_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Gamma_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right. \\
& \left.\left. - \frac{\ln R_2/R_1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)} + \frac{n^2 - \mu_y'}{P_2}\right]\right\} + n \cot^4 \alpha\left\{S_{32,1}\left(\Lambda_1\Gamma_1 + \Lambda_2\Gamma_2 + \Lambda_3\Gamma_3 + \Lambda_4\Gamma_4\right.\right.
\end{aligned}
$$

(Equation continued on next page)

$$
- \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\Bigg) + S_{32,2}\,\frac{R_2}{R_1}\left[\Lambda_1\Gamma_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Gamma_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Gamma_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Gamma_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right.
$$

$$
\left. + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\right]\Bigg\} + \cot^2\alpha\left(S_{34,1} + S_{34,2}\right)
$$

$$
\gamma_{3m} = \frac{B_\theta}{B_y}\cot^4\alpha\Bigg[S_{31,1}\Bigg(\Phi_{1m}\psi_1 + \Phi_{2m}\psi_2 + \Phi_{3m}\psi_3 + \Phi_{4m}\psi_4
$$

$$
- \frac{\frac{m\pi}{\ln R_2/R_1}\left\{\mu'_y\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg) + S_{31,2}\,\frac{R_2}{R_1}\Bigg(\Phi_{1m}\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Phi_{2m}\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1}
$$

$$
+ \Phi_{3m}\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Phi_{4m}\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{(-1)^m\,\frac{m\pi}{\ln R_2/R_1}\left\{\mu'_y\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg)\Bigg]
$$

$$
+ n\cot^4\alpha\Bigg\{S_{32,1}\Big[\Lambda_1\Phi_{1m} + \Lambda_2\Phi_{2m} + \Lambda_3\Phi_{3m} + \Lambda_4\Phi_{4m}
$$

(Equation continued on next page)

$$- \frac{\frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg] + S_{32,2}\,\frac{R_2}{R_1}\left[\Lambda_1\Phi_{1m}\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Phi_{2m}\left(\frac{R_2}{R_1}\right)^{-\lambda_1}\right.$$

$$\left. + \Lambda_3\Phi_{3m}\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Phi_{4m}\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{(-1)^m\,\frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right]$$

$$\left. + \frac{m\pi}{\ln R_2/R_1}\cot^2\alpha\left[S_{34,1} + (-1)^m S_{34,2}\right]\right\}$$

$$\gamma_4 = \frac{R_2}{R_1}\ln\frac{R_2}{R_1}\cot^3\alpha\; S_{33,2} + S_{31,2}\,\frac{R_2}{R_1}\ln\frac{R_2}{R_1}\cot^4\alpha\;\frac{B_\theta}{B_y}\left[\Omega_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Omega_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1}\right.$$

$$\left. + \Omega_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Omega_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)}\right] + S_{41,1}\cot^3\alpha\;\frac{B_\theta}{B_y}\left[\Omega_1\psi_1 + \Omega_2\psi_2\right.$$

$$\left. + \Omega_3\psi_3 + \Omega_4\psi_4 + \frac{1}{\left(B_\theta/B_y\right)\left(1 - n^2\right)}\right] + nS_{42,1}\cot^3\alpha\left(\Lambda_1\Omega_1 + \Lambda_2\Omega_2 + \Lambda_3\Omega_3 + \Lambda_4\Omega_4\right.$$

$$\left. + \frac{1}{n^2 - 1}\right) + nS_{32,2}\,\frac{R_2}{R_1}\ln\frac{R_2}{R_1}\cot^4\alpha\left[\Lambda_1\Omega_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Omega_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Omega_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}\right.$$

(Equation continued on next page)

$$+ \Lambda_4\Omega_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{1}{n^2 - 1}\Bigg] + S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y}\Bigg[\Omega_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Omega_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Omega_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}$$

$$+ \Omega_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{1}{\left(B_\theta/B_y\right)\left(1 - n^2\right)}\Bigg] + nS_{42,2} \cot^3 \alpha\Bigg[\Lambda_1\Omega_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Omega_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1}$$

$$+ \Lambda_3\Omega_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Omega_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{1}{n^2 - 1}\Bigg] + \cot^2 \alpha\left(S_{43,1} + S_{41,2}\right)$$

$$\gamma_5 = \frac{R_2}{R_1}\left(\ln \frac{R_2}{R_1}\right)^2 \cot^3 \alpha\ S_{33,2} + S_{31,2} \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \frac{B_\theta}{B_y}\Bigg[\Gamma_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Gamma_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1}$$

$$+ \Gamma_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Gamma_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{\ln R_2/R_1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)} + \frac{n^2 - \mu'_y}{P_2}\Bigg] + \cot \alpha\left[S_{44,1} + \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2}\right]$$

$$+ \cot^3 \alpha \frac{B_\theta}{B_y}\Bigg\{S_{41,1}\left(\Gamma_1\psi_1 + \Gamma_2\psi_2 + \Gamma_3\psi_3 + \Gamma_4\psi_4 + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2}\right) + S_{41,2}\Bigg[\Gamma_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1}$$

$$+ \Gamma_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Gamma_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Gamma_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{\ln R_2/R_1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)} + \frac{n^2 - \frac{\mu'_\theta}{B_\theta/B_y}}{P_2}\Bigg]\Bigg\}$$

$$+ n \cot^3 \alpha\Bigg\{S_{42,1}\left[\Lambda_1\Gamma_1 + \Lambda_2\Gamma_2 + \Lambda_3\Gamma_3 + \Lambda_4\Gamma_4 - \frac{1 - \frac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\right] + S_{42,2}\Bigg[\Lambda_1\Gamma_1\left(\frac{R_2}{R_1}\right)^{\lambda_1}$$

(Equation continued on next page)

$$+ \Lambda_2\Gamma_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Gamma_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Gamma_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\Bigg]\Bigg\}$$

$$+ nS_{32,2} \cot\alpha\Bigg[\Lambda_1\Gamma_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Gamma_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Gamma_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Gamma_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1}$$

$$- \frac{1 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\Bigg] + \ln\frac{R_2}{R_1}\cot^2\alpha\left(S_{34,2} + S_{43,2}\right)$$

$$\gamma_{6m} = S_{31,1}\frac{R_2}{R_1}\ln\frac{R_2}{R_1}\cot^4\alpha\,\frac{B_\theta}{B_y}\Bigg(\Phi_{1m}\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Phi_{2m}\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Phi_{3m}\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}$$

$$+ \Phi_{4m}\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{(-1)^m\dfrac{m\pi}{\ln R_2/R_1}\left\{\mu'_y\left[\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\dfrac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg)$$

$$+ \frac{m\pi}{\ln R_2/R_1}\cot\alpha\left[S_{44,1} + (-1)^m\left(\frac{R_2}{R_1}\right)^{-1} S_{44,2}\right] + \cot^3\alpha\,\frac{B_\theta}{B_y}\Bigg[S_{41,1}\Bigg(\Phi_{1m}\psi_1 + \Phi_{2m}\psi_2$$

(Equation continued on next page)

$$+ \Phi_{3m}\psi_3 + \Phi_{4m}\psi_4 - \frac{\frac{m\pi}{\ln R_2/R_1}\left\{\frac{\mu'_\theta}{B_\theta/B_y}\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg)$$

$$+ S_{41,2}\left(\Phi_{1m}\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Phi_{2m}\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Phi_{3m}\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Phi_{4m}\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right.$$

$$\left. - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1}\left\{\frac{\mu'_\theta}{B_\theta/B_y}\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right)\Bigg] + n \cot^3 \alpha \Bigg\{ S_{42,1}\Bigg[\Lambda_1\Phi_{1m} + \Lambda_2\Phi_{2m}$$

$$+ \Lambda_3\Phi_{3m} + \Lambda_4\Phi_{4m} - \frac{\frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg] + S_{42,2}\Bigg[\Lambda_1\Phi_{1m}\left(\frac{R_2}{R_1}\right)^{\lambda_1}$$

$$+ \Lambda_2\Phi_{2m}\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Phi_{3m}\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Phi_{4m}\left(\frac{R_2}{R_1}\right)^{-\lambda_2}$$

$$- \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg]\Bigg\} + n\frac{R_2}{R_1}\ln\frac{R_2}{R_1} S_{32,2} \cot^4 \alpha \Bigg[\Lambda_1\Phi_{1m}\left(\frac{R_2}{R_1}\right)^{\lambda_1}$$

(Equation continued on next page)

$$+ \Lambda_2\Phi_{2m}\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Phi_{3m}\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Phi_{4m}\left(\frac{R_2}{R_1}\right)^{-\lambda_2}$$

$$\left. - \frac{(-1)^m\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right] + S_{34,2} \cot^2 \alpha \frac{m\pi}{\ln R_2/R_1} (-1)^m \ln \frac{R_2}{R_1}$$

$$\gamma_{7p} = \frac{p\pi}{x_1}\left(S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y}\left[\Omega_1\psi_1 + \Omega_2\psi_2 + \Omega_3\psi_3 + \Omega_4\psi_4 + \frac{1}{\left(B_\theta/B_y\right)\left(1 - n^2\right)}\right]\right.$$

$$+ nS_{42,1} \cot^3 \alpha \left(\Lambda_1\Omega_1 + \Lambda_2\Omega_2 + \Lambda_3\Omega_3 + \Lambda_4\Omega_4 + \frac{1}{n^2 - 1}\right) + S_{43,1} \cot^2 \alpha$$

$$+ (-1)^p \left\{S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y}\left[\Omega_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Omega_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Omega_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Omega_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right.\right.$$

$$\left. + \frac{1}{\left(B_\theta/B_y\right)\left(1 - n^2\right)}\right] + nS_{42,2} \cot^3 \alpha \left[\Lambda_1\Omega_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Omega_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Omega_3\left(\frac{R_2}{R_1}\right)^{\lambda_2}\right.$$

$$\left.\left. + \Lambda_4\Omega_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{1}{n^2 - 1}\right] + S_{43,2} \cot^2 \alpha\right\}\Bigg)$$

$$
\begin{aligned}
\gamma_{8p} = \frac{p\pi}{x_1}\Bigg(& S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y}\left(\Gamma_1\psi_1 + \Gamma_2\psi_2 + \Gamma_3\psi_3 + \Gamma_4\psi_4 + \frac{n^2 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{P_2}\right) \\
&+ nS_{42,1} \cot^3 \alpha \left[\Lambda_1\Gamma_1 + \Lambda_2\Gamma_2 + \Lambda_3\Gamma_3 + \Lambda_4\Gamma_4 - \frac{1 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\right] + S_{44,1} \cot \alpha \\
&+ (-1)^p \Bigg\{ S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y}\Bigg[\Gamma_1\psi_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Gamma_2\psi_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Gamma_3\psi_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Gamma_4\psi_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} \\
&- \frac{\ln R_2/R_1}{\left(B_\theta/B_y\right)\left(n^2 - 1\right)} + \frac{n^2 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{P_2}\Bigg] + nS_{42,2} \cot^3 \alpha \Bigg[\Lambda_1\Gamma_1\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Gamma_2\left(\frac{R_2}{R_1}\right)^{-\lambda_1} \\
&+ \Lambda_3\Gamma_3\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Gamma_4\left(\frac{R_2}{R_1}\right)^{-\lambda_2} + \frac{\ln R_2/R_1}{n^2 - 1} - \frac{1 - \dfrac{\mu'_\theta}{B_\theta/B_y}}{\left(n^2 - 1\right)^2}\Bigg] + S_{43,2} \cot^2 \alpha \ln \frac{R_2}{R_1} \\
&+ \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2} \cot \alpha \Bigg\}\Bigg)
\end{aligned}
$$

$$\gamma_{9mp} = \frac{p\pi}{x_1}\left\{S_{41,1} \cot^3 \alpha \frac{B_\theta}{B_y}\left(\Phi_{1m}\psi_1 + \Phi_{2m}\psi_2 + \Phi_{3m}\psi_3 + \Phi_{4m}\psi_4\right.\right.$$

$$\left.- \frac{\frac{m\pi}{\ln R_2/R_1}\left\{\frac{\mu'_\theta}{B_\theta/B_y}\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right) + nS_{42,1} \cot^3 \alpha \left[\Lambda_1\Phi_{1m} + \Lambda_2\Phi_{2m}\right.$$

$$\left.+ \Lambda_3\Phi_{3m} + \Lambda_4\Phi_{4m} - \frac{\frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu'_\theta\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right] + S_{44,1} \cot \alpha \frac{m\pi}{\ln R_2/R_1}$$

$$+ (-1)^p\left[S_{41,2} \cot^3 \alpha \frac{B_\theta}{B_y}\left(\psi_1\Phi_{1m}\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \psi_2\Phi_{2m}\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \psi_3\Phi_{3m}\left(\frac{R_2}{R_1}\right)^{\lambda_2}\right.\right.$$

$$\left.\left.+ \psi_4\Phi_{4m}\left(\frac{R_2}{R_1}\right)^{-\lambda_2} - \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1}\left\{\frac{\mu'_\theta}{B_\theta/B_y}\left[\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + 1\right] - n^2\right\}}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\right)\right]$$

$$+ nS_{42,2} \cot^3 \alpha \left[\Lambda_1\Phi_{1m}\left(\frac{R_2}{R_1}\right)^{\lambda_1} + \Lambda_2\Phi_{2m}\left(\frac{R_2}{R_1}\right)^{-\lambda_1} + \Lambda_3\Phi_{3m}\left(\frac{R_2}{R_1}\right)^{\lambda_2} + \Lambda_4\Phi_{4m}\left(\frac{R_2}{R_1}\right)^{-\lambda_2}\right.$$

(Equation continued on next page)

$$- \frac{(-1)^m \frac{m\pi}{\ln R_2/R_1}\left(\frac{B_\theta}{B_y} - \mu_\theta'\right)}{\left(\frac{m\pi}{\ln R_2/R_1}\right)^4 - P_1\left(\frac{m\pi}{\ln R_2/R_1}\right)^2 + P_2}\Bigg] + \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2} \cot \alpha \frac{m\pi}{\ln R_2/R_1} (-1)^{p+m}\Bigg\}$$

$\gamma_{9p} = \gamma_{9mp}$ with $m = p$

The quantities β are defined as $\beta = \gamma$ with S_{ij} replaced by $M_{ij} \sin^4 \alpha$.

For symmetric deformations, the quantities t_i are replaced by t_{oi} defined as

$$t_{o1} = \Omega_{o1}\left[\left(\frac{R_2}{R_1}\right)^{\lambda_o} - 1\right] + \Omega_{o2}\left[\left(\frac{R_2}{R_1}\right)^{-\lambda_o} - 1\right]$$

$t_{o2} = t_{o1}$

with Ω_{oi} replaced by Γ_{oi}.

$t_{o3} = t_{o1}$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$- \frac{\frac{m\pi}{x_1}\left[1 - (-1)^m\right]}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)}$$

$$t_{o4} = \frac{\Omega_{o1}}{\lambda_o}\left[1 + \left(\frac{R_2}{R_1}\right)^{\lambda_o}\left(\lambda_o \ln \frac{R_2}{R_1} - 1\right)\right] - \frac{\Omega_{o2}}{\lambda_o}\left[1 - \left(\frac{R_2}{R_1}\right)^{-\lambda_o}\left(\lambda_o \ln \frac{R_2}{R_1} + 1\right)\right]$$

$t_{o5} = t_{o4}$

with Ω_{oi} replaced by Γ_{oi}.

$t_{o6m} = t_{o4}$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$+ \frac{\frac{m\pi}{x_1}(-1)^m \ln R_2/R_1}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)}$$

$$t_{o7p} = \frac{\lambda_o \frac{p\pi}{x_1}}{\left(\frac{p\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \left\{\Omega_{o1}\left[1 - (-1)^p\left(\frac{R_2}{R_1}\right)^{\lambda_o}\right] - \Omega_{o2}\left[1 - (-1)^p\left(\frac{R_2}{R_1}\right)^{-\lambda_o}\right]\right\}$$

$t_{o8p} = t_{o7p}$

with Ω_{oi} replaced by Γ_{oi}.

$t_{o9mp} = t_{o7p}$

with Ω_{oi} replaced by Φ_{oim}.

$t_{o9p} = t_{o7p}$

with Ω_{oi} replaced by Φ_{oim} and the additional term

$$- \frac{\frac{1}{2}\left(\frac{p\pi}{x_1}\right)^2 \ln R_2/R_1}{\left(\frac{p\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)}$$

For symmetric deformation, the quantities γ_i are replaced by γ_{oi} defined as

$$\gamma_{o1} = \cot^4 \alpha \left\{ S_{13,2}\left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) + 1\right] \right.$$

$$\left. + \frac{R_2}{R_1} S_{13,2}\left[\Omega_{o1}\left(\frac{R_2}{R_1}\right)^{\lambda_o}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Omega_{o2}\left(\frac{R_2}{R_1}\right)^{-\lambda_o}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) + 1\right]\right\}$$

$$+ \cot^3 \alpha \left(S_{33,1} + \frac{R_2}{R_1} S_{33,2}\right)$$

$$\gamma_{o2} = \cot^4 \alpha \left\{ S_{13,1} \left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \mu'_y \right] \right.$$

$$\left. + \frac{R_2}{R_1} S_{13,2} \left[\Gamma_{o1}\left(\frac{R_2}{R_1}\right)^{\lambda_o}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Gamma_{o2}\left(\frac{R_2}{R_1}\right)^{-\lambda_o}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) + \ln\frac{R_2}{R_1} - \mu'_y \right] \right\}$$

$$+ \frac{R_2}{R_1} \ln\frac{R_2}{R_1} S_{33,2} \cot^3 \alpha + \cot^2 \alpha \left(S_{34,1} + S_{34,2}\right)$$

$$\gamma_{o3m} = \cot^4 \alpha \left\{ \frac{R_2}{R_1} + S_{31,2} \left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right. \right.$$

$$\left. - \frac{\mu'_\theta \frac{m\pi}{x_1}(-1)^m}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] + S_{31,1} \left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) \right.$$

$$\left. \left. - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] \right\} + \frac{m\pi}{x_1} \cot^2 \alpha \left[S_{34,1} + (-1)^m S_{34,2} \right]$$

$$\gamma_{o4} = \cot^3 \alpha \left\{ S_{14,1} \left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) + 1 \right] \right.$$

$$\left. + S_{14,2} \left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} + 1 \right] \right\}$$

$$+ \cot^2 \alpha \left(S_{34,1} + S_{34,2}\right) + \frac{R_2}{R_1} \ln\frac{R_2}{R_1} \left\{ S_{13,2} \cot^4 \alpha \left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} \right. \right.$$

$$\left. \left. - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} + 1 \right] + S_{33,2} \cot^3 \alpha \right\}$$

$$\gamma_{o5} = \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \left\{ S_{13,2} \cot^4 \alpha \left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right.\right.$$

$$\left.\left. + \ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right] + S_{33,2} \ln \frac{R_2}{R_1} \cot^3 \alpha \right\} + S_{34,2} \cot^2 \alpha \ln \frac{R_2}{R_1}$$

$$+ S_{41,2} \cot^3 \alpha \left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right.$$

$$\left. + \ln \frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y} \right] + S_{43,2} \ln \frac{R_2}{R_1} \cot^2 \alpha + \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2} \cot \alpha + S_{44,1} \cot \alpha$$

$$+ S_{41,1} \cot^3 \alpha \left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Gamma_{02}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \frac{\mu'_\theta}{B_\theta/B_y} \right]$$

$$\gamma_{o6m} = \frac{R_2}{R_1} \ln \frac{R_2}{R_1} \cot^4 \alpha \; S_{13,2} \left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right.$$

$$\left. - \frac{\mu'_\theta \frac{m\pi}{x_1}(-1)^m}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] + \ln \frac{R_2}{R_1} \cot^2 \alpha \frac{m\pi}{x_1}(-1)^m S_{34,2} + \cot^3 \alpha \left\{ S_{14,1} \left[\Phi_{o1m} \right.\right.$$

$$\left. \times \left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] + S_{14,2} \left[\Phi_{o1m} \right.$$

$$\left.\left. \times \left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} - \frac{\mu'_\theta \frac{m\pi}{x_1}(-1)^m}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] \right\}$$

$$+ \frac{m\pi}{x_1} \cot \alpha \left[S_{44,1} + (-1)^m \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2} \right]$$

$$\gamma_{o7p} = \frac{p\pi}{x_1}\left(\cot^3\alpha\left\{S_{14,1}\left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) + 1\right] + (-1)^p S_{14,2}\right.\right.$$

$$\left.\times\left[\Omega_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Omega_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} + 1\right]\right\}$$

$$\left. + \cot^2\alpha\left[S_{34,1} + (-1)^p S_{34,2}\right]\right)$$

$$\gamma_{o8p} = \frac{p\pi}{x_1}\left(\cot^3\alpha\left\{S_{14,1}\left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \frac{\mu'_\theta}{B_\theta/B_y}\right]\right.\right.$$

$$+ (-1)^p S_{14,2}\left[\Gamma_{o1}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Gamma_{o2}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o}\right.$$

$$\left.\left. + \ln\frac{R_2}{R_1} - \frac{\mu'_\theta}{B_\theta/B_y}\right]\right\} + (-1)^p \ln\frac{R_2}{R_1}\cot^2\alpha\, S_{34,2} + \cot\alpha\left[S_{44,1} + (-1)^p\left(\frac{R_2}{R_1}\right)^{-1} S_{44,2}\right]\Bigg)$$

$$\gamma_{o9mp} = \frac{p\pi}{x_1}\cot^3\alpha\left\{S_{14,1}\left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \frac{\mu'_\theta\frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)}\right]\right.$$

$$+ (-1)^p S_{14,2}\left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o}\right.$$

$$\left.\left. - \frac{\mu'_\theta\frac{m\pi}{x_1}(-1)^m}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)}\right] + \frac{p\pi}{x_1}\frac{m\pi}{x_1}\cot\alpha\left[S_{44,1} + (-1)^{m+p}\left(\frac{R_2}{R_1}\right)^{-1} S_{44,2}\right]\right\}$$

$$\gamma_{o9p} = \frac{m\pi}{x_1} \cot^3 \alpha \left\{ S_{14,1} \left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right) - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right) - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] \right.$$

$$+ (-1)^m S_{14,2} \left[\Phi_{o1m}\left(\sqrt{B_\theta/B_y} - \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{\lambda_o} - \Phi_{o2m}\left(\sqrt{B_\theta/B_y} + \mu'_\theta\right)\left(\frac{R_2}{R_1}\right)^{-\lambda_o} \right.$$

$$\left. \left. - \frac{\mu'_\theta \frac{m\pi}{x_1}}{\left(\frac{m\pi}{x_1}\right)^2 + \left(B_\theta/B_y\right)} \right] \right\} + \left(\frac{m\pi}{x_1}\right)^2 \cot \alpha \left[S_{44,1} + \left(\frac{R_2}{R_1}\right)^{-1} S_{44,2} \right]$$

For symmetric deformations, the quantities β_i are replaced by β_{oi} defined as

$\beta_{oi} = \gamma_{oi}$

with S_{ij} replaced by $M_{ij} \sin^4 \alpha$.

EXPRESSIONS FOR S_{ij} AND M_{ij}

The expressions for S_{ij} are

$$S_{11} = \bar{S}_{11} \cos^2 \alpha + 2\bar{S}_{13} \sin \alpha \cos \alpha + \bar{S}_{33} \sin^2 \alpha$$

$$S_{12} = S_{21} = \bar{S}_{12} \cos \alpha + \bar{S}_{23} \sin \alpha$$

$$S_{13} = S_{31} = \left(\sin^2 \alpha - \cos^2 \alpha\right)\bar{S}_{13} + \left(\bar{S}_{11} - \bar{S}_{33}\right) \sin \alpha \cos \alpha$$

$$S_{14} = S_{41} = \bar{S}_{14} \cos \alpha + \bar{S}_{34} \sin \alpha$$

$$S_{22} = \bar{S}_{22}$$

$$S_{23} = S_{32} = -\bar{S}_{23} \cos \alpha + \bar{S}_{12} \sin \alpha$$

$$S_{24} = S_{42} = \bar{S}_{24}$$

$$S_{33} = \bar{S}_{11} \sin^2 \alpha + \bar{S}_{33} \cos^2 \alpha - 2\bar{S}_{13} \sin \alpha \cos \alpha$$

$$S_{34} = S_{43} = -\bar{S}_{34} \cos \alpha + \bar{S}_{14} \sin \alpha$$

$$S_{44} = \bar{S}_{44}$$

$$\bar{S}_{11} = \frac{\eta^2\left(R_1/R\right)^2 Z\bar{A}}{\bar{z}_o}\left\{\frac{\left(R_1/R\right)^2}{\bar{z}_o^2}\left[\frac{\eta^2\bar{I}_z}{\cos \alpha} + \frac{\bar{J}}{2\left(1 + \mu_r\right) \cos \alpha} - 2\eta^2 \frac{z_o}{R} \tan \alpha \, \bar{I}_{yz} + \eta^2\left(\frac{z_o}{R}\right)^2 \tan \alpha \sin \alpha \, \bar{I}_y\right] + \eta^2\left(\frac{z_o}{R}\right)^2 \tan \alpha \sin \alpha\right\}$$

$$\bar{S}_{12} = \eta^3\left(\frac{R_1}{R}\right)^2 Z\bar{A}\left[\frac{\left(R_1/R\right)^2}{\bar{z}_o^2 \cos \alpha}\left(\bar{I}_{yz} - \frac{z_o}{R} \sin \alpha \, \bar{I}_y\right) - \frac{z_o}{R} \tan \alpha\right]$$

$$\bar{S}_{13} \; \frac{\eta^2\left(R_1/R\right)^2 Z\bar{A}}{\bar{z}_o}\left[\eta^2 \, \frac{\left(R_1/R\right)^2}{\bar{z}_o}\left(\frac{\bar{I}_{yz}}{\cos \alpha} - \tan \alpha \, \frac{z_o}{R} \bar{I}_y\right) - \tan \alpha \, \frac{z_o}{R}\left(1 + \eta^2 \, \frac{z_o}{R} \cos \alpha\right)\right]$$

$$\bar{S}_{14} = \frac{\eta^2 (R_1/R)^3 Z\bar{A}}{\bar{z}_o^2} \left\{ \left(\eta^2 \frac{z_o}{R} \frac{\cos\alpha}{\bar{z}_o} - 1 \right) \bar{I}_z + \frac{1}{2(1+\mu_r)} \left(\frac{z_o}{R} \frac{\cos\alpha}{\bar{z}_o} - 1 \right) \bar{J} \right.$$

$$\left. + \frac{z_o}{R} \sin\alpha \left[\frac{\eta^2}{\bar{z}_o} \left(1 - \frac{z_o}{R} \cos\alpha \right) + 1 \right] \bar{I}_{yz} - \eta^2 \left(\frac{z_o}{R} \right)^2 \frac{\sin^2\alpha}{\bar{z}_o} \bar{I}_y \right\} - \eta^2 \left(\frac{z_o}{R} \right)^2 \frac{R_1}{R} \frac{\sin^2\alpha}{\bar{z}_o} Z\bar{A}$$

$$\bar{S}_{22} = \frac{\eta^2 \bar{z}_o (R_1/R)^2 Z\bar{A}}{\cos\alpha} \left[1 + \frac{(R_1/R)^2}{\bar{z}_o^2} \bar{I}_y \right]$$

$$\bar{S}_{23} = \frac{\eta (R_1/R)^2 Z\bar{A}}{\cos\alpha} \left[\eta^2 \frac{(R_1/R)^2}{\bar{z}_o} \bar{I}_y + 1 + \eta^2 \frac{z_o}{R} \cos\alpha \right]$$

$$\bar{S}_{24} = \eta Z\bar{A} \left[\left(\frac{R_1}{R} \right)^3 \frac{1}{\bar{z}_o} \left(\frac{\eta^2}{\bar{z}_o} \frac{z_o}{R} \cos\alpha - 1 \right) \bar{I}_{yz} + \frac{z_o}{R} \sin\alpha \frac{R_1}{R} + \eta^2 \frac{z_o}{R} \frac{\sin\alpha}{\bar{z}_o^2} \left(\frac{R_1}{R} \right)^3 \bar{I}_y \right]$$

$$\bar{S}_{33} = \frac{(R_1/R)^2 Z\bar{A}}{\bar{z}_o \cos\alpha} \left[\left(1 + \eta^2 \cos\alpha \frac{z_o}{R} \right)^2 + \eta^4 \left(\frac{R_1}{R} \right)^2 \bar{I}_y \right]$$

$$\bar{S}_{34} = \frac{\eta^2 (R_1/R)^3 Z\bar{A}}{\bar{z}_o} \left[\left(\frac{\eta^2}{\bar{z}_o} \frac{z_o}{R} \cos\alpha - 1 \right) \bar{I}_{yz} + \eta^2 \frac{z_o}{R} \frac{\sin\alpha}{\bar{z}_o} \bar{I}_y \right] + \frac{\sin\alpha}{\bar{z}_o} \frac{z_o}{R} \frac{R_1}{R} Z\bar{A} \left(1 + \eta^2 \frac{z_o}{R} \cos\alpha \right)$$

$$\bar{S}_{44} = \frac{\cos\alpha\, Z\bar{A}}{\bar{z}_o} \left\{ \left(\frac{R_1}{R} \right)^2 \left[\left(1 - \frac{\eta^2}{\bar{z}_o} \frac{z_o}{R} \cos\alpha \right)^2 \bar{I}_z + \frac{2\eta^2}{\bar{z}_o} \frac{z_o}{R} \sin\alpha \left(\frac{\eta^2}{\bar{z}_o} \frac{z_o}{R} \cos\alpha - 1 \right) \bar{I}_{yz} \right. \right.$$

$$\left. \left. + \frac{\eta^2 \left(1 - \frac{1}{\bar{z}_o} \frac{z_o}{R} \cos\alpha \right)^2}{2(1+\mu_r)} \bar{J} + \eta^4 \left(\frac{z_o}{R} \right)^2 \frac{\sin^2\alpha}{\bar{z}_o^2} \bar{I}_y \right] + \left(\frac{z_o}{R} \right)^2 \sin^2\alpha \right\}$$

where z_o is the eccentricity of ring centroidal axis measured from inside shell surface, positive for external ring, and

$$\bar{A} = \frac{E_r A_r}{R_1 B_\theta}$$

$$\bar{I}_y = \frac{I_z \cos^2 \alpha + I_y \sin^2 \alpha + 2I_{yz} \sin \alpha \cos \alpha}{R_1^2 A_r}$$

$$\bar{I}_z = \frac{I_y \cos^2 \alpha + I_z \sin^2 \alpha - 2I_{yz} \sin \alpha \cos \alpha}{R_1^2 A_r}$$

$$\bar{I}_{yz} = \frac{I_{yz}\left(\sin^2 \alpha - \cos^2 \alpha\right) + \sin \alpha \cos \alpha\left(I_z - I_y\right)}{R_1^2 A_r}$$

$$\bar{J} = \frac{J}{R_1^2 A_r}$$

$$\bar{z}_o = 1 + \frac{z_o}{R} \cos \alpha$$

The expressions for M_{ij} are

$$M_{11} = \overline{M}_{11} \cos^2 \alpha + 2\overline{M}_{13} \sin \alpha \cos \alpha + \overline{M}_{33} \sin^2 \alpha$$

$$M_{12} = M_{21} = \overline{M}_{12} \cos \alpha + \overline{M}_{23} \sin \alpha$$

$$M_{13} = M_{31} = \left(\sin^2 \alpha - \cos^2 \alpha\right)\overline{M}_{13} + \left(\overline{M}_{11} - \overline{M}_{33}\right)\sin \alpha \cos \alpha$$

$$M_{14} = M_{41} = \overline{M}_{14} \cos \alpha + \overline{M}_{34} \sin \alpha$$

$$M_{22} = \overline{M}_{22}$$

$$M_{23} = M_{32} = -\overline{M}_{23} \cos \alpha + \overline{M}_{12} \sin \alpha$$

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$$M_{33} = \overline{M}_{11} \sin^2 \alpha + \overline{M}_{33} \cos^2 \alpha - 2\overline{M}_{13} \sin \alpha \cos \alpha$$

$$M_{34} = M_{43} = -\overline{M}_{34} \cos \alpha + \overline{M}_{14} \sin \alpha$$

$M_{44} = \overline{M}_{44}$

where

$$\overline{M}_{11} = \bar{\rho} \cos \alpha \left\{ \bar{z}_o \left[1 + \eta^2 \left(\frac{z_o}{R}\right)^2 \sin^2 \alpha\right] + \eta^2 \left(\frac{z_o}{R}\right)^2 \left(\frac{R_1}{R}\right)^2 \frac{\sin^2 \alpha}{\bar{z}_o} \bar{I}_y + \eta^2 \left(\frac{R_1}{R}\right)^2 \frac{\bar{I}_z}{\bar{z}_o} \right.$$

$$\left. - 2\eta^2 \frac{z_o}{R} \frac{\sin \alpha}{\bar{z}_o} \left(\frac{R_1}{R}\right)^2 \bar{I}_{yz} \right\}$$

$$\overline{M}_{12} = \bar{\rho} \cos \alpha \left[-\eta \left(\frac{z_o}{R}\right) \sin \alpha \, \bar{z}_o^2 - \eta \frac{z_o}{R} \left(\frac{R_1}{R}\right)^2 \sin \alpha \, \bar{I}_y + \eta \left(\frac{R_1}{R}\right)^2 \bar{I}_{yz}\right]$$

$$\overline{M}_{13} = \bar{\rho} \cos \alpha \left[-\eta^2 \left(\frac{z_o}{R}\right)^2 \bar{z}_o \sin \alpha \cos \alpha - \eta^2 \left(\frac{z_o}{R}\right) \sin \alpha \left(\frac{R_1}{R}\right)^2 \bar{I}_y + \eta^2 \left(\frac{R_1}{R}\right)^2 \bar{I}_{yz}\right]$$

$$\overline{M}_{14} = \bar{\rho} \cos^2 \alpha \left[\left(\frac{R_1}{R}\right)^{-1} \left(\frac{z_o}{R}\right) \cos \alpha \, \bar{z}_o - \eta^2 \frac{R_1}{R} \left(\frac{z_o}{R}\right)^2 \frac{\sin^2 \alpha}{\bar{z}_o} \bar{I}_y + \eta^2 \frac{\cos \alpha}{\bar{z}_o} \left(\frac{z_o}{R}\right) \frac{R_1}{R} \bar{I}_z \right.$$

$$\left. + \eta^2 \left(\frac{z_o}{R}\right) \frac{R_1}{R} \frac{\sin \alpha}{\bar{z}_o} \left(1 - \frac{z_o}{R} \cos \alpha\right) \bar{I}_{yz}\right]$$

$$\overline{M}_{22} = \bar{\rho} \bar{z}_o \cos \alpha \left[\bar{z}_o^2 + \left(\frac{R_1}{R}\right)^2 \bar{I}_y\right]$$

$$\overline{M}_{23} = \eta \bar{\rho} \bar{z}_o \cos \alpha \left[\frac{z_o}{R} \bar{z}_o \cos \alpha + \left(\frac{R_1}{R}\right)^2 \bar{I}_y\right]$$

$$\overline{M}_{24} = \eta \bar{\rho} \frac{z_o}{R} \cos^2 \alpha \frac{R_1}{R} \left(\bar{I}_y \sin \alpha + \bar{I}_{yz} \cos \alpha\right)$$

$$\overline{M}_{33} = \bar{\rho} \cos \alpha \bar{z}_o \left[1 + \eta^2 \left(\frac{z_o}{R}\right)^2 \cos^2 \alpha + \eta^2 \left(\frac{R_1}{R}\right)^2 \bar{I}_y\right]$$

$$\overline{M}_{34} = \bar{\rho}\cos^2\alpha\left(\frac{R_1}{R}\right)^{-1}\left(\frac{z_o}{R}\right)\left[\sin\alpha\,\bar{z}_o + \eta^2\sin\alpha\left(\frac{R_1}{R}\right)^2\bar{I}_y + \eta^2\cos\alpha\left(\frac{R_1}{R}\right)^2\bar{I}_{yz}\right]$$

$$\overline{M}_{44} = \bar{\rho}\left(\frac{R_1}{R}\right)^{-2}\cos^3\alpha\left(\bar{z}_o\left(\frac{z_o}{R}\right)^2 + \frac{\left(R_1/R\right)^2}{\bar{z}_o}\left\{\bar{I}_y\left[\bar{z}_o^2 + \eta^2\left(\frac{z_o}{R}\right)^2\sin^2\alpha\right]\right.\right.$$

$$\left.\left. + \bar{I}_z\left[\bar{z}_o^2 + \eta^2\left(\frac{z_o}{R}\right)^2\cos^2\alpha\right] + 2\left(\frac{z_o}{R}\right)^2\sin\alpha\cos\alpha\,\eta^2\bar{I}_{yz}\right\}\right)$$

$$\bar{\rho} = \frac{\rho A_r}{\gamma R_1}$$

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